STRENGTH OF MATERIALS

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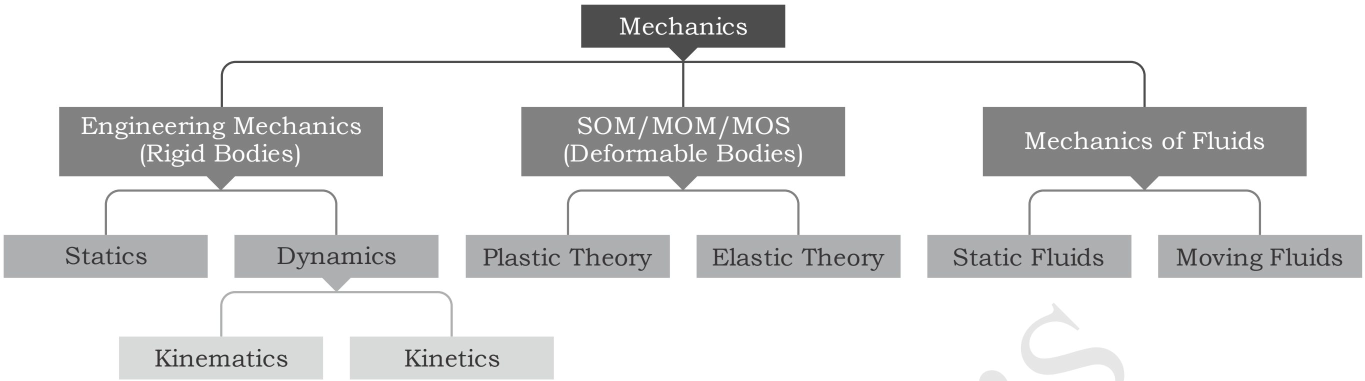
SIMPLE STRESSES & STRAINS

INTRODUCTION



Mechanics : It is the branch of science deals with both internal & external forces and their effect on the structure.

- *Engineering Mechanics* : It deals with external forces & their effect on the rigid bodies.
  - *Statics* : It is the branch of engineering mechanics deals with the bodies under rest condition (or) moving with constant velocity.
  - *Dynamics* : It is the branch of engineering mechanics deals with the bodies under motion.
    - *Kinematics* : It is the branch of dynamics deals with the bodies under motion without considering the forces (or) energies causing it.
    - *Kinetics* : It is the branch of dynamics deals with the bodies under motion by considering the forces (or) energies causing it.
- *Strength of Materials* : It deals with the internal forces caused due to external forces & their effect on the deformable bodies.
  - *Theory of elasticity* : This theory is valid when the material is under elastic condition.
  - *Theory of plasticity* : This theory is valid when the material is under plastic condition.

<u>Aim of SOM</u> : It's aim is to derive the relation between stresses and strains and to find out the angle of obliquity.

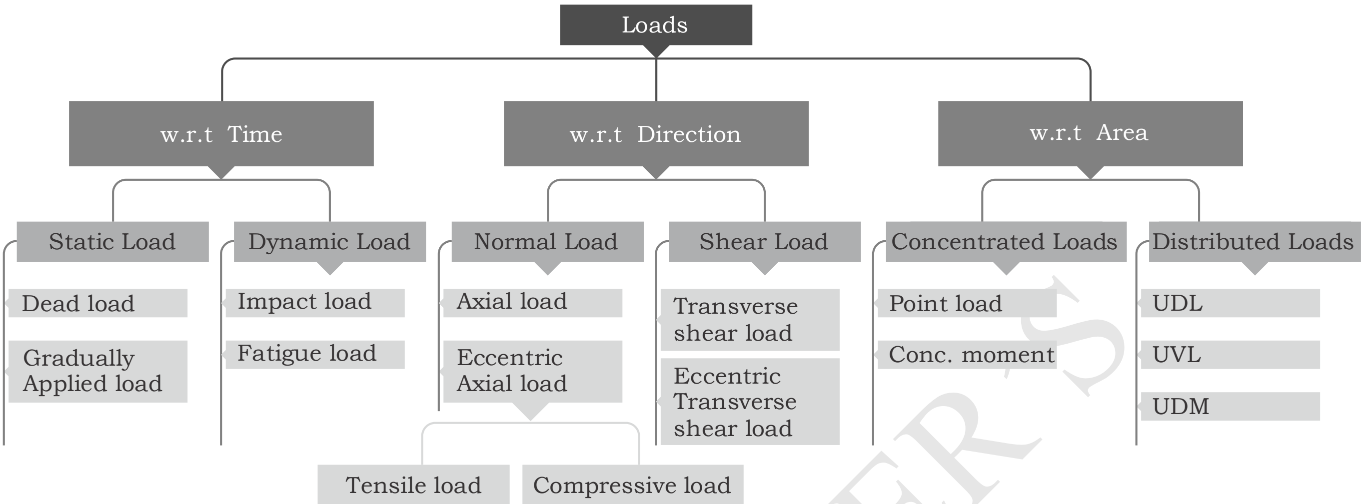
Assumptions

- The material should be homogeneous and isotropic.
- The member should follow Hooke's law.
- The loading on a member should be static.
- The member should be prismatic.
- Self-weight of the member can be neglected.
- The material should be continuous.
- Super position principle must be valid.

  Super position principle is not valid for:
  - Deep Beams
  - Sinking of supports
  - Long Columns
  - Torsion on Non-circular member's
- Saint-venant's principle must be valid.

LOAD Load is a component acts externally on the other component during its function.

Classification of loads:



w.r.t TIME

Static Load: The load does not vary w.r.t time (if there is any variation time has no effect on it), is known as static load.

- *<u>Dead Load:</u>* Loads due to non-living bodies are considered to be dead loads. Eg: Self-weight of the building members
- *<u>Gradually applied loads:</u>* The loads which vary gradually with respect to time and remains constant when it reaches to maximum value.

Dynamic Loads: The load varies with respect to time (Time effect need to be considered) is known as dynamic load.

- *<u>Impact Load:</u>* If any load is falling from some height then it is know as impact load
  - The load act within fraction of time known as Impact load.
  - If we have any doubt whether the loading is static (or) impact, it is always better to consider it as impact load, because
  - $\sigma_{impact} > \sigma_{static}$

    $\sigma_{impact} = I.F \times \sigma_{static}$ $\boxed{I.F = 1 + \sqrt{1 + \frac{2h}{\delta_{static}}}}$

    $\sigma_{impact} \geq 2\sigma_{Static}$

    I.F. $\rightarrow$ Impact factor

    h $\rightarrow$ height / depth of fall

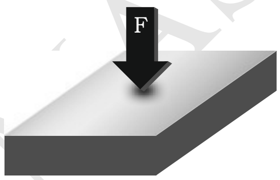
    δ_{static} $\rightarrow$ deformation due to static loads.
- *<u>Fatigue:</u>* The deerease in strength of the material due to repetative loads is known as fatigue.

- The loads acting as number of cycles (or) number of repetitions on material are known as fatigue loads. These loads may change according to direction (or) load intensity (or) both direction & load intensity.
- *Endurance limit/fatigue limit:* It is a stress value, below which under application of any reversal of stresses ,the material has high probability of no failure.
- Endurance limit for different materials

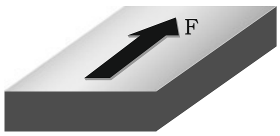
Material	Endurance Limit
Mildsteel	186 MPa
Iron	165 MPa
Aluminum	131 MPa
Copper	96 MPa

w.r.t DIRECTION

Normal Loads: The loads acting normal (or) perpendicular to the surface are called normal loads.



Shear Loads: The loads acting parallel (or) tangential to the surface are called shear loads.



STRESS It is an internal resistance force acting at a point that resists deformation due to external load (or) the total internal resistance force acting per unit area.

$$\sigma = \frac{\text{Resistance Force}}{\text{Area}} = \frac{R}{A}$$

- *Pressure:* Pressure is defined as externally applied normal force at a point
  - always acts normal to the surface or towards the surface.

$$P = \frac{\text{Applied Normal Force / Thrust}}{\text{Area}} = \frac{F}{A}$$

- *Units:* N/mm^2 (or) KSC (or) PSI

- *Strength:* It is the limiting (or) maximum stress allowed in a material without causing any failure (or) fracture.

Note :

- For externally applied force on a material first strain develops then followed by stress.
- For Equilibrium condition we consider resistence force = applied force in stress formula.
- If material is free to move even under application of small external load then there is no internal resistance ($\sigma = 0$).

STRAIN The material is said to be strained, if the relative position of the particles is changed (or) altered.

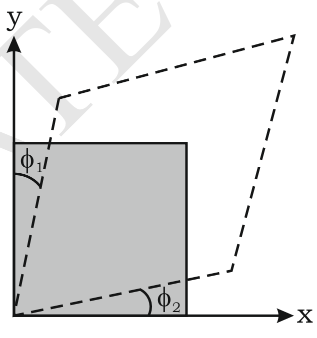
Normal Strain: The ratio of change in dimensions to the original dimensions.

$$\varepsilon = \frac{\text{Change in Dimensions}}{\text{Original Dimensions}}$$ Units : No units

- *Longitudinal Strain:* If strain occurs along the direction of the applied load, it is known as longitudinal strain .
- *Lateral Strain:* If strain occurs perpendicular to the direction of the applied load, it is known as lateral strain.
- Generally, One longitudinal strain is associated with two lateral strains.
- *Volumetric Strain:* The ratio of change in volume to the original volume is known as volumetric strain.

$$\varepsilon_V = \frac{\delta_V}{V} = \frac{V_F - V_i}{V_i}$$

Shear Strain: The angular change between two mutually perpendicular planes.



$\phi = \phi_1 + \phi_2$

Unit : Radian

MECHANICAL PROPERTIES

Strength: It is the ability of the material to resist the extenal load without failure.

Stiffness: It is the ability of the material to resist the deformation.

Elasticity: It is the ability of the material to deform under the load application of and regains to its original shape when the load is removed.

Plasticity: It is the ability of the material to deform under the application of load and does not returns to its original shape when the load is removed.

Ductility: It is the ability of the material which can deform plastically under tensile load.

Malleability: It is the ability of the material which can deform plastically without rupture under compressive load.

Resilience: It is the ability of the material to absorb energy when deformed elastically and to release it when unloaded.

Toughness: It is the ability of the material to absorb energy up to breaking point.

Brittleness: It is the property of the material fails suddenly without any visible deformation.

Fatigue: Failure of the material under repetative and reversible stresses.

Creep: Slow and progressive deformation with long term stress under a constant load.

STRESS - STRAIN CURVES

- Load versus Extension curve can be drawn by using UTM (Universal Testing Machine) results.
- Extensometer is used to find the extension in bar.

$$\sigma = \frac{\text{Load}}{\text{Area}}$$

$$\varepsilon = \frac{\text{Change in Gauge Length}}{\text{Original Gauge Length}}$$

AS per ASTM standards

$$\boxed{\text{Gauge Length} = 5.65\sqrt{A_0}}$$ A_0 = Area of cross-section with in gauge length

- Gauge length only depends upon the cross sectional dimensions and it is independent on length, shape, type of material and loading on material.
- As per U.T.M machine,the relation between engineering stress and actual stress is given as :

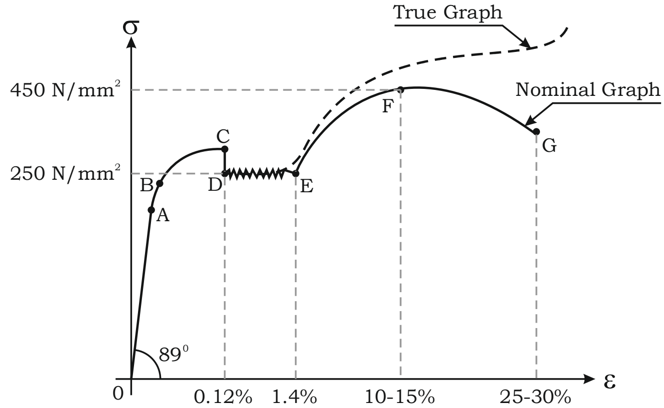
$$\boxed{\sigma_a = \sigma_0(1 \pm \varepsilon_0)} \quad \boxed{\varepsilon_a = \ln(1 \pm \varepsilon_0)}$$

σ_a = Actual stress

σ_0 = Nominal stress / Engineering stress

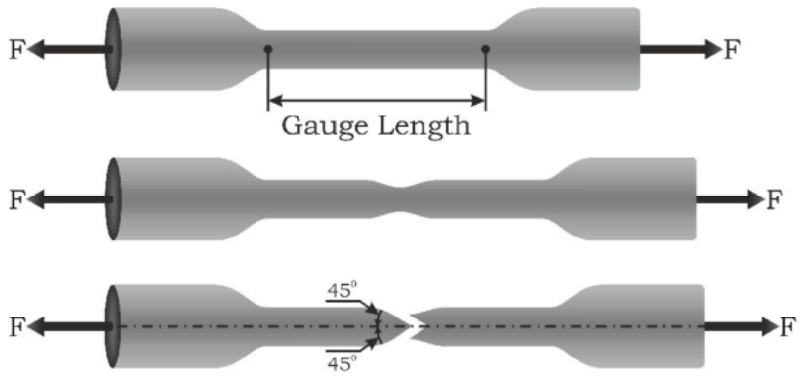
ε_0 = Nominal strain

Stress Strain Curve for Mild Steel In Tension



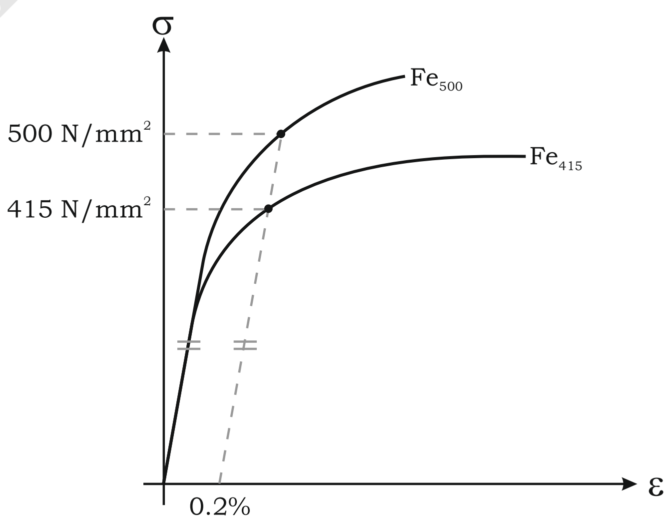
POINTS	ZONES
O ⟶ Origin	OB ⟶ Elastic Zone
A ⟶ Proportionality limit	OA ⟶ Linear elastic zone
B ⟶ Elastic limit	AB ⟶ Non linear elastic zone
C ⟶ Upper yield point	BC ⟶ Elasto-plastic zone
D ⟶ Lower yield point	CD ⟶ Yield zone
E ⟶ Plastic limit	DE ⟶ Plastic zone / Yield plateau
F ⟶ Ultimate point	EF ⟶ Strain hardening zone
G ⟶ Breaking point	EG ⟶ Strain softening zone

Failure Criteria:



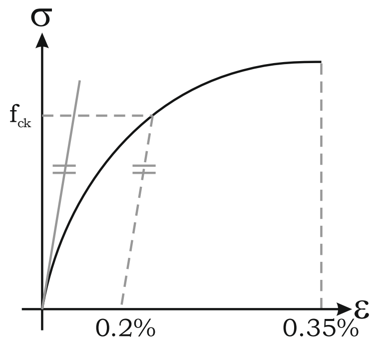
Offset Method:

- In case of Aluminium, iron, steel etc. it is hard to locate yield point. For locating the yield point offset method is adopted.
- At 0.2% of strain, draw the parallel line to the linear elastic line, the line will meelt the curve curve at a point, the corresponding stress value is considered as proof stress (or) yield stress.
- For non-linear elastic curves, draw initial tangent and draw the parallel line to initial tangent at 0.2% of strain, where it meets the curve, at that point the corresponding stress value is considered as proof stress (or) yield stress.



Stress Strain Curve for Concrete

- Brittle material must be tested in compression.



- In case of brittle materials, offset method is used to find out the first crack in the material.

Ductility index (or) Ductility factor: $\frac{\varepsilon_{Fail}}{\varepsilon_{Yield}}$

D.I = 1.75 → for Concrete

D.I = 200 to 250 → for Mild Steel

Note :

- Ductile materials are strong in tension, moderate in compression and weak in shear.
- Brittle materials are weak in tension, moderate in shear and strong in compression.

Hooke's Law: It states that stress is directly proportional to strain up to proportionality limit

Factor of saftey: It is the ratio of maximum permissible stress to working stress.

ELASTIC CONSTANTS

Elastic constants are known as material properties within elastic zone.

Elastic Modulus: [E]

$$E = \frac{\text{Normal Stress}}{\text{Linear Strain}}$$

$$E = \frac{\sigma}{\varepsilon}$$

Units: N/mm^2

E → Young's modulus (or) Modulus of Elasticity (or) Elastic Modulus.

Shear Modulus: [C, N or G]

$$G = \frac{\text{Shear Stress}}{\text{Shear Strain}}$$

$$G = \frac{\tau}{\phi}$$

Units: $\frac{N/mm^2}{Radians}$

G $\rightarrow$ Shear modulus (or) rigidity modulus (or) modulus of rigidity.

Bulk Modulus: [K]

$$K = \frac{\text{Average Normal Stress}}{\text{Volumetric Strain}}$$

$$K = \frac{\sigma}{\varepsilon_v}$$

Units: N/mm^2

K $\rightarrow$ Bulk modulus (or) Dilatant constant

Poisson's Ratio: [μ or 1/m or ν]

$$\mu = \frac{\text{-Lateral Strain}}{\text{Longitudinal Strain}}$$

- Range of Poisson's ratio : -1 to 0.5
  - For genetic material : -1 to 0
  - For engineering Material : 0 to 0.5
  - For incompressible material : μ=0.5

    Eg:-Water, Rubber, Saturated Clay, Paraffin wax Etc..,

Relation between elastic constants

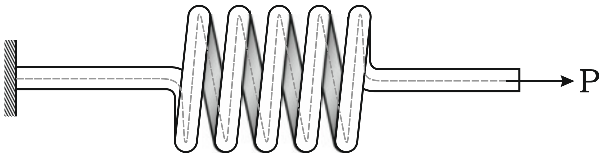
Relations between E,G, μ	E = 2G (1+ μ)
Relations between E,K, μ	E = 3K (1-2μ)
Relations between K,G, μ	$\mu = \frac{3K-2G}{6K+2G}$
Relations between E,G,K	$E = \frac{9KG}{3K+G}$

- For an isotropic material, $\boxed{E > K > G}$

Material	Total Number of Elastic Constants	Total Number of Independent Elastic Constants
Isotropic	4	2
Orthotropic	12	9
Non Isotropic	∞	21

AXIAL DEFORMATIONS

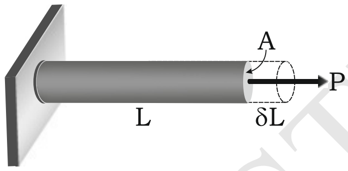
Stiffness: It is the resistance of a material against to the external deformation and it is represented by "k".



$k = \frac{P}{\delta}$ $k \rightarrow$ Stiffness of spring

$\delta = \frac{P}{k}$ $1/k \rightarrow f \rightarrow$ Flexibility of spring

Units:- N/mm



$$\delta L = \frac{PL}{AE}$$

Limitations of axial deformations:

- The material must be homogeneous, isotropic and prismatic.
- Loading must be uni-axial and constant.
- Loading must be within proportionality limit.

STRESSES AND DEFORMATIONS DUE TO SELF WEIGHT

- Maximum Stress due to self weight in prismatic bar : $\sigma_{sw} = \gamma L$
- Total Deformation due to self-weight in prismatic bar : $\delta L = \frac{\gamma L^2}{2E}$
- Deformation due to external load: $\delta = \frac{WL}{AE}$
- Deformation due to self weight $\delta_{\text{self weight}} = \frac{1}{2}\frac{WL}{AE}$

Note :

- Deformation due to self weight is equal to half of the deformation due to external load of same load intensity
- Incase of prismatic bar stresses and deformations due to self-weight depends only on length of the bar and material properties but independent on area of cross-section & shape of the material.

Stresses and Deformations in conical bar due to self weight:

- Deformation due to self weight in case of conical bar is: $\delta L = \frac{\gamma L^2}{6E}$
- Stress due to self weight in case of conical bar is: $\sigma_{sw} = \frac{\gamma L}{3}$

Bar of Uniform Strength

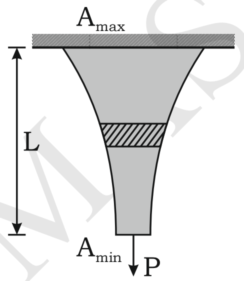
- If a bar is subjected to equal axial stresses at all points in a material then the bar is said to be bar of uniform strength.

  Ex: Weightless prismatic bar subjected to external load 'W' at its end.
- But in practice self weight is also considered.
- If self weight is acting on bar to make it as a bar of uniform strength, the bar variation must follow the equation of $\frac{A_{max}}{A_{min}} = e^{\frac{\gamma L}{\sigma}}$.

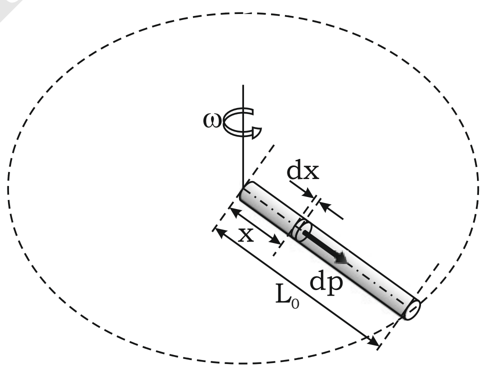
  L → Length of bar

  γ → Weight density of material

  σ → Stress in the material



Stresses and Deformations in a bar due to Centrifugal force



Maximum normal stress in rod : $\sigma_{max} = \frac{\rho\omega^2 L_0^{\,2}}{2}$

Total change in length in rod : $\boxed{\Delta L = \dfrac{\rho \omega^2 L_0^{\,3}}{3E}}$

ρ = Mass density of material

ω = Angular speed

L_0 = Length of the rod

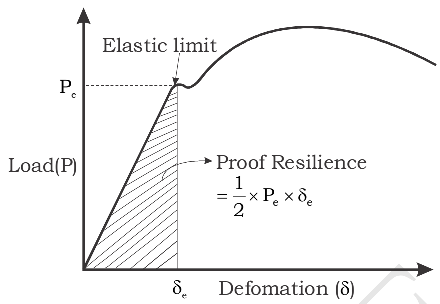
E = Elastic modulus

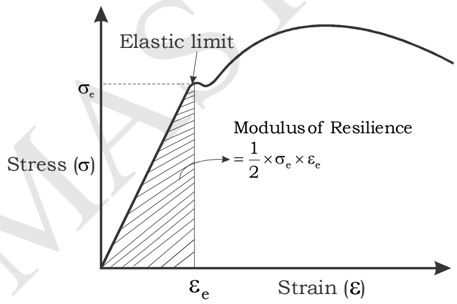
STRAIN ENERGY

- A body develops internal forces to resist any external forces acting on it.
- Work is also done by the internal forces due to deformation of the body this is known as the internal work (or) strain energy.
- Internal energy stored in a material due to external work done is known as strain energy.

Resilience: It is strain energy stored in a material up to Elastic limit.

Proof Resilience: It is maximum strain energy stored in a material up to Elastic limit.



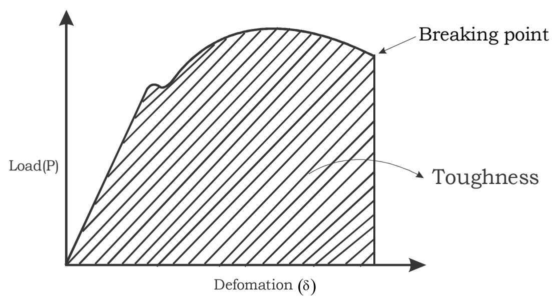


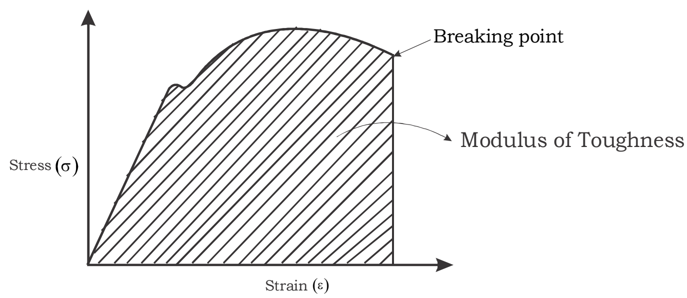
Modulus of Resilience: It is maximum strain energy stored in a material up to Elastic limit per unit volume.

> **Note :** Modulus of Resilience is constant for a given material like youngs modulus and poission's ratio.

Toughness:

- It is strain energy stored in a material up to Breaking point.





Modulus of Toughness:

It is strain energy stored in a material up to Breaking point per unit volume.

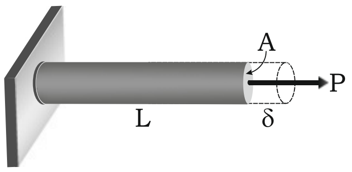
Note : Ductile materials are more tough than brittle materials.

Strain Energy Stored In a Axially Loaded Bar

Strain energy stored in a bar due to Axial force (P) is $U = \int_0^L \frac{P_x^2 dx}{2AE}$

AE = Axial Rigidity

- **Bar is subjected to the Axial load:**



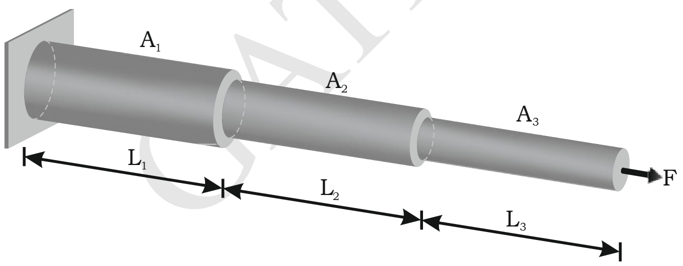
$$U = \frac{1}{2} \times P \times \delta$$

$$U = \frac{P^2 L}{2AE}$$

$$U = \frac{1}{2} \times \sigma \times \varepsilon \times \text{volume}$$

$$U = \frac{\sigma^2}{2E} \times \text{volume}$$

- **Stepped bar is subjected to the Axial load:**

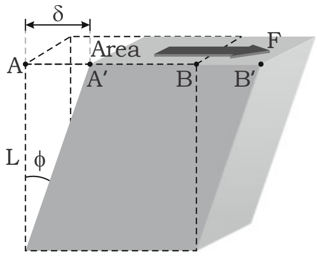


$$U = \frac{P^2}{2E} \sum_{i=1}^{n} \left[\frac{L_i}{A_i} \right]$$

- **Strain Energy stored in a bar due to the self weight:**

$$U = \frac{\gamma^2 A L^3}{6E}$$

- **Strain Energy stored in a member due to the shear force:**



$$\boxed{U = \frac{\tau^2}{2G} \times \text{volume}}$$

- **Strain energy stored in a beam due to the flexural stresses**

$$\boxed{U = \frac{f^2}{2E} \times \text{volume}}$$

f=flexural stress
E=Young's modulus

Stresses Due to Various Types of loading

The load can be applied to the body in three different ways

- Gradually applied load
- Suddenly applied load
- Impact (or) Falling load

- **Gradually applied load:**

Work done = Average load × Extension

$$= \frac{1}{2} P \times \delta$$

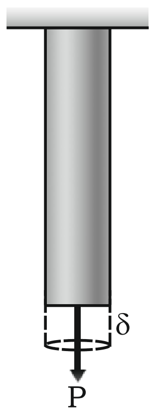
Strain Energy, $U = \frac{\sigma^2}{2E} \times AL$

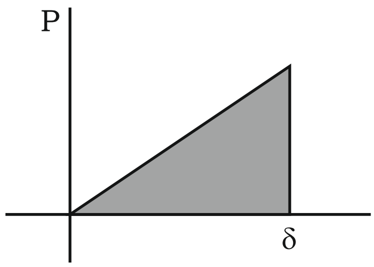
From law of conservation of energy

W.D=U

$$\frac{1}{2} \times P \times \delta = \frac{\sigma^2}{2E} \times AL$$

$$\boxed{\sigma = \frac{P}{A}}$$





Note : It is the general expression for stress. In all the problems unless otherwise specified load is always considered to gradually applied.

- **Suddenly Applied load:**

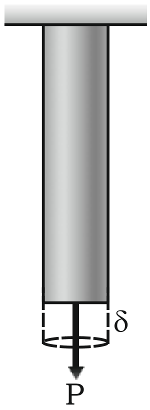
Work done by load =load × extension

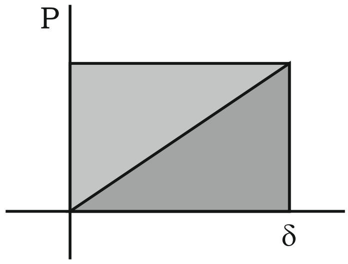
Strain energy stored $U = \frac{\sigma^2}{2E} \times AL$

From law of conservation of energy W.D=U

$$P \times \delta = \frac{\sigma^2}{2E} \times A \times L$$

$$\boxed{\sigma = \frac{2P}{A}}$$





Note : Stress induced in case of suddenly applied load is twice that when the load is applied gradually

- **Impact or Falling Load:**

Work done by load = P(h+ δ)

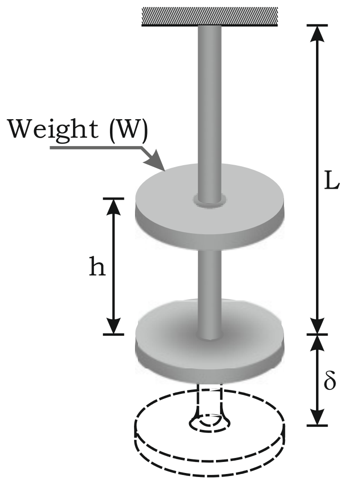
Strain energy stored, $U = \frac{\sigma^2}{2E} \times AL$

From law of consenation of energy W.D=U

$$P(h + \delta) = \frac{\sigma^2}{2E} \times AL$$

By upon simplification above equation finally we get

$$\sigma_{impact} = \sigma_{static}\left[1 + \sqrt{1 + \frac{2h}{\delta_{static}}}\right] \qquad \because \ \delta_{static} = \frac{PL}{AE}$$



$$\sigma_{impact} = \sigma_{Static} \times \text{Impact factor}$$

$$\boxed{\sigma_{impact} = \sigma_{Static} \times I.F}$$

TEMPERATURE STRESSES/THERMAL STRESSES

- The stresses developed due to change in temperature are known as temperature stresses (or) thermal stresses.
- These are the secondary stresses.
- The stresses due to external loads are primary stresses.
- The temperature stresses can be reduced by using same material (or) by using diffrent materials with same co-efficent of thermal expansion(α).
- Coefficient of thermal expansion is constant for a particular material like E & μ
- α = Coefficient of thermal expansion (or) $\dfrac{\text{strain}}{\text{unit degree change in temperature}}$

$\alpha = \dfrac{\varepsilon}{t} \Rightarrow \boxed{\varepsilon = \alpha t} \rightarrow$ Thermal strain

$\delta L = L\alpha t$

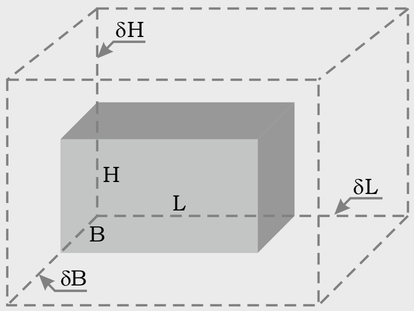
$\sigma_t = E \times \varepsilon \Rightarrow \sigma_t = E\alpha t \rightarrow$ Thermal stress

$t \rightarrow$ Change in temperature

- **Coefficient of thermal expansion for different materials**

$\alpha \rightarrow$ steel	: $12 \times 10^{-6}/^0c$
$\alpha \rightarrow$ brass	: $19 \times 10^{-6}/^0c$
$\alpha \rightarrow$ diamond	: $1 \times 10^{-6}/^0c$
$\alpha \rightarrow$ copper	: $17 \times 10^{-6}/^0c$
$\alpha \rightarrow$ glass	: $8.5 \times 10^{-6}/^0c$
$\alpha \rightarrow$ aluminium	: $21.1 \times 10^{-6}/^0c$
$\alpha \rightarrow$ water	: $65 \times 10^{-6}/^0c$
$\alpha \rightarrow$ concrete	: $12 \times 10^{-6}/^0c$
$\alpha \rightarrow$ iron	: $0.2 \times 10^{-6}/^0c$
$\alpha \rightarrow$ gold	: $14 \times 10^{-6}/^0c$

<table>
<tr><th>Case - I
Material is free to expand</th><th>Case - II
Expansion is restricted</th><th>Case - III
Expansion allowed by</th></tr>
<tr>
<td>
If material subjected to rise of temperature t^0C



$\delta B = B\alpha t$

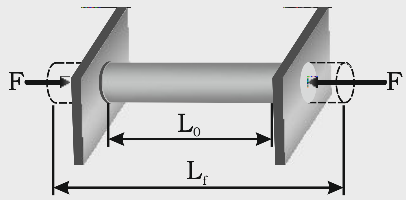
$\delta L = L\alpha t$

$\delta H = H\alpha t$

No thermal stress develops in the material when it is free to expand

$\sigma_t = 0$

$\varepsilon_t = \alpha t$
</td>
<td>
If material subjected to rise of temperature t^0C



Free expansion = expansion that is restricted

$L_0 \alpha t = \frac{PL_0}{AE}$

$\alpha t = \frac{P}{AE}$

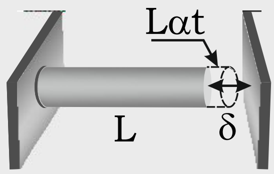
$\sigma_t = E.\alpha.t$

$\varepsilon_t = 0$

If the expansion is restricted, thermal stresses will develops in the material.
</td>
<td>
If material subjected to rise of temperature t^0C

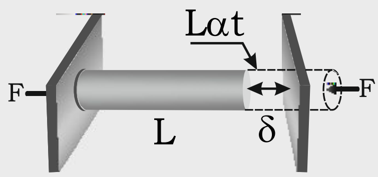
If $(L\alpha t \leq \delta)$

i.e. material is free to expand



No stress is developed in the material

$\varepsilon_t = \alpha t$



If $(L\alpha t > \delta)$

$((L\alpha t) - \delta) = \frac{PL}{AE}$

$\sigma_t = \frac{(L\alpha t - \delta)E}{L}$

$\varepsilon_t = \frac{\delta L}{L}$
</td>
</tr>
</table>